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Is Re_θ/M_e a Meaningful Transition Criterion?

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Introduction

THE stability and transition of boundary layers at supersonic and hypersonic speeds has been studied by the research community for more than 50 years. Much has been learned about the physics underlying the transition process through years of analysis, experiment, and computation leading to physics-based methods of transition Reynolds number estimation. This wealth of information, however, has often been ignored by the vehicle-design community. Rather, they have tended to rely on empirical transition correlations of questionable basis and reliability and have thus deprived themselves of dealing with transition constructively and imaginatively.

Many in the vehicle-design community commit considerable computational fluid dynamics resources to flowfield computation, but they then expect the transition Reynolds number to be determined from simple algebraic correlations. But how can one expect a phenomenon that is rooted in the *unsteady* Navier–Stokes equations to be governed by a simple formula? Rather than use the correlations, they should incorporate physics-based modules into their flowfield computation as done, for example, in the Hyper-X/X-43 design.

A popular design-transition criterion is Re_θ/M_e equal to a constant or some function of other variables. Most often, there is not enough reliable information about the effects of other variables on which to base correlating factors. Thus, the criterion is usually $Re_\theta/M_e = \text{const}$. Also, because Re_θ varies as the square root of a length Reynolds number, any scatter in Re_θ is greatly amplified when inverted to get a length-transition Reynolds number or a length to transition. There is no apparent physics basis for this form of transition correlation. Nevertheless, the validity of Re_θ/M_e correlations will be tested by recasting physics-based results in Re_θ/M_e terms and examining whether they make sense. The cases to be considered are transition due to roughness on a flat plate at supersonic speeds, roughness-induced transition on spherical nosetips, and flight-transition data for sharp cones.

Transition is the response of the laminar boundary layer to forcing environmental disturbances. The signature of these disturbances in the boundary layer is determined by the receptivity process. Those disturbances to which the boundary layer is sensitive are then amplified by either eigenmode-growth (the familiar and well-documented Tollmien–Schlichting (T–S), crossflow, or Görtler mechanisms) or by transient-growth mechanisms. Transient growth arises through the nonorthogonal nature of the Orr–Sommerfeld and Squire eigenfunctions [1,2]. The largest effects come from the superposition of slightly damped, highly oblique (near streamwise) T–S and Squire modes. Transient growth is subcritical with respect to the T–S neutral curve and so precedes T–S growth or occurs when T–S growth is absent. The transient-growth signature is essentially algebraic growth followed by exponential decay. Transient growth is therefore a candidate mechanism for many examples of "bypass transition."

Roughness-Induced Transition on a Flat Plate

As part of their study of transient growth applied to roughness-induced transition, Reshotko and Tumin [3] computed the transient-growth factors for a flat plate in supersonic flow. The growth factor is a function of both Mach number and surface-temperature level and scales with length Reynolds number. These are incorporated into a transition model similar to that of Andersson et al. [4] for freestream-turbulence effects on transition. We assume that an energy norm at transition is related to an input energy through the transient growth factor G:

$$E_{\text{tr}} = GE_{\text{in}} \tag{1}$$

The input energy is in the form of a density times velocity squared, where the roughness-induced-disturbance velocities are assumed proportional to the roughness height k. The momentum thickness θ is chosen as the reference length because it is the least sensitive to surface-temperature level of any of the boundary-layer scales. The resulting expression for E_{in} is

$$E_{\text{in}} = (\rho_w/\rho_e)(k/\theta)^2 \tag{2}$$

which, for a boundary layer, can be approximated as

$$E_{\text{in}} = (T_e/T_w)(k/\theta)^2 \tag{3}$$

The growth factor G scales with the length Reynolds number or the square of a thickness Reynolds number. Thus, from Eqs. (1) and (3), we can write

$$(E_{\text{tr}})^{1/2} = (G^{1/2}/Re_\theta)Re_\theta(k/\theta)(T_e/T_w)^{1/2} \tag{4}$$

where $G^{1/2}/Re_\theta$ can be obtained from Fig. 1 of [3] using the following relation for a flat plate with $\mu \sim T$ and $Pr = 1$:

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$$(G^{1/2}/Re_\theta) = 1.506(G/Re_L)^{1/2} \tag{5}$$

Assuming that transition occurs when E_{tr} reaches some constant level, $Re_{\theta,\text{tr}}$ can be written

$$Re_{\theta,\text{tr}} = \text{const}(G^{1/2}/Re_\theta)^{-1}(k/\theta)^{-1}(T_w/T_e)^{1/2} \tag{6}$$

or

$$Re_{\theta,\text{tr}}(k/\theta) = U_e k/\nu_e = \text{const}(G^{1/2}/Re_\theta)^{-1} \times (T_w/T_{\text{aw}})^{1/2}\{1 + r[(\gamma-1)/2]M^2\} \tag{7}$$

In the absence of better information, the const in Eq. (6) is evaluated from the experimental results of Feindt [5], who found that $U_e k/\nu_e$ is about 120 for incompressible flow with zero pressure gradient. Because $G^{1/2}/Re_\theta = 0.1021$ for incompressible flat plate flow, const $= 12.25$.

Figure 1 shows $Re_{\theta,\text{tr}}(k/\theta)/M_e = U_e k/(\nu_e M_e)$ vs Mach number. This is as close to Re_θ/M_e as the formulation allows. Note the dependence on surface-temperature level and the inherent dependence on roughness height.

The upsweep in the curves as the Mach number decreases below two is the beginning of a $1/M_e$ behavior. A more revealing figure is obtained when not dividing the ordinates by M_e. This is shown in Fig. 2. Note that eliminating the $1/M_e$ behavior yields straight lines. Note also that for T_w/T_{aw} just below 0.25, the line would go through the origin so that $Re_{\theta,\text{tr}}(k/\theta)/M_e$ would be a constant. Were roughness data to be correlated only with Re_θ/M_e, one could expect lots of scatter because of the omission of k/θ and surface-temperature information.

Based on the preceding results for flat plates, it is recommended that roughness-induced transition data for other configurations be plotted as in Fig. 2, $U_e k/\nu_e$ vs M_e, with wall-temperature isotherms indicated. This will allow the relevant physics to be included in the correlations.

To be noted is that cooling is destabilizing for transient growth, whereas for T–S disturbances, cooling stabilizes the first mode but destabilizes the second mode.

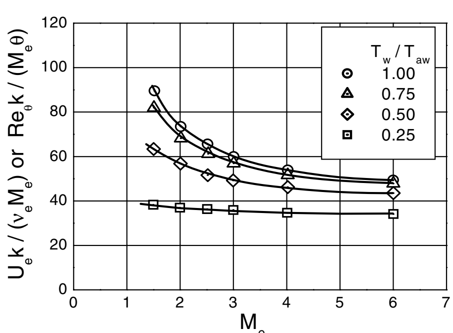

Fig. 1 $\boldsymbol{Re_{\theta,\text{tr}}(k/\theta)/M_e = U_e k/(\nu_e M_e)}$ **vs** $\boldsymbol{M_e}$**.**

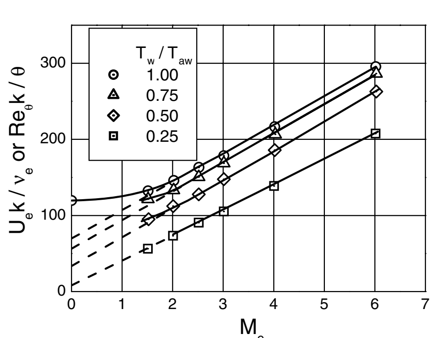

Fig. 2 Variation of roughness parameter $\boldsymbol{U_e k/\nu_e}$ **with Mach number and surface-temperature level.**

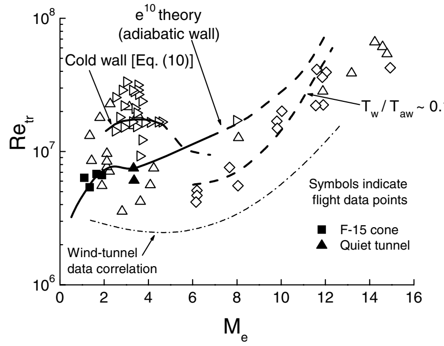

Fig. 3 Transition on sharp cones.

Roughness-Induced Transition on Spherical Nosetips

Extensive transient-growth calculations were carried out [3] for axisymmetric stagnation-point flows. These are relevant to the spherical nosetip of hypersonic sphere-cone configurations, for which there is an extensive experimental database and significant transition correlations (Batt and Legner [6,7] and Reda [8,9]). A transition model such as that described earlier for the flat plate was developed [3] and incorporated the transient growth results. This resulted in the following relation:

$$Re_{\theta,\text{tr}} = 180(k/\theta)^{-1}(2T_w/T_e)^{1.27} \tag{8}$$

In the preceding relation, the k^{-1} behavior comes from the model assumption, whereas the $T_w^{1.27}$ behavior comes from the transient-growth results. Because the transient-growth theory is linear, the numerical factor of 180 has to come from a data set. Reference [3] demonstrates that this relation correlates the passive ablative nosetip data (PANT) [6,7] and the Reda [8,9] data. The agreement is both qualitative and quantitative. Because θ appears in the numerator of both sides of Eq. (8), this relation can be rewritten as

$$U_e k/\nu_e = 180(2T_w/T_e)^{1.27} \tag{9}$$

The left side of Eq. (9) is the same as Reda's [8,9] $Re_{ke,\text{tr}}$. For $T_w/T_{\text{aw}} = 0.33$, Eq. (9) gives Reda's value of 106. Reda estimates his surface-temperature level to have been about 0.3. Note further that the correlating expression does not contain the Mach number M_e; hence, Re_θ/M_e has no meaning in this case.

Transition on Smooth, Sharp Cones

There is an extensive database of supersonic flight data for smooth, sharp cones, as shown in Fig. 3. Included also are some NASA Langley Research Center quiet-tunnel results. The data are for various surface-thermal conditions. Also shown on the figure are the calculations by Malik [10] using an e^N method that shows what is expected for adiabatic wall conditions as well as cooled conditions. The solid lines on the chart show where the first mode is dominant and the dashed lines show where the second mode is dominant.

The calculations of [10] are up to $M = 7$ for the adiabatic and cooled walls and the $M = 6$ point for the cold wall. The second mode curves beyond $M = 7$ are by an unknown extrapolator. Actual calculations beyond $M = 6$ depend on the gas model, the chemistry assumptions, and whether one uses parallel flow or parabolized stability equation (PSE) methods. Some results for $M > 6$ recently provided to this author by Malik† for both perfect gas and with chemistry show the same trends as in Fig. 3 but differ somewhat numerically. Those differences do not affect the conclusions reached in this paper and so they are not further elaborated upon herein.

Because the data are well-represented by Malik's e^N calculations, these e^N lines will now be plotted as Re_θ vs M_e in Fig. 4 and as Re_θ/M_e vs M_e in Fig. 5. The cold-wall curves are for

$$T_w/T_{\text{aw}} = 1\text{–}0.05M_e - 0.0025M_e^2 \tag{10}$$

†Private communications with M. R. Malik, January and February 2007.

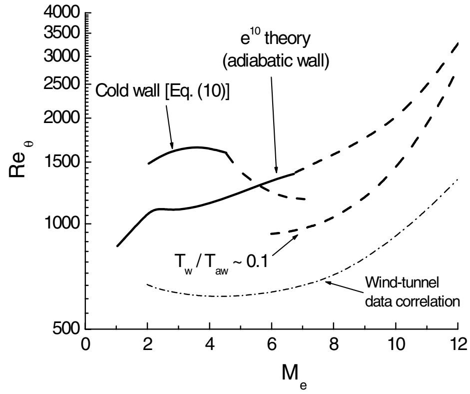

Fig. 4 Transition on sharp cones, Re_θ vs M_e.

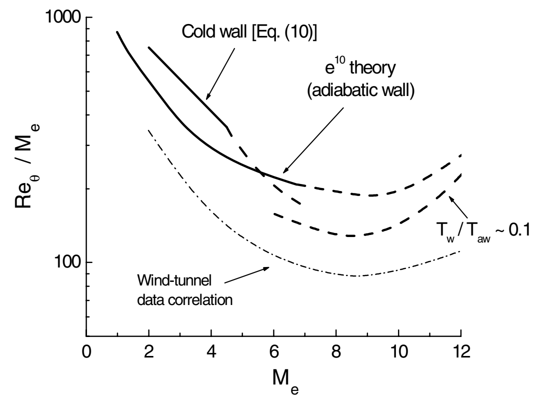

Fig. 5 Transition on sharp cones, Re_θ/M_e vs M_e.

which equals 0.8 at $M_e = 3.4$. The curves in Fig. 4 look like those in Fig. 3, except that the ordinate range was collapsed from two orders of magnitude in Fig. 3 to one order of magnitude in Fig. 4. Figure 5 differs from Fig. 4 only insofar as one sees the $1/M_e$ upsweep for the lower Mach numbers, as also mentioned in conjunction with Fig. 1. No particular correlational revelations appear in Fig. 5, whereas, as mentioned earlier, the physics-based e^N calculations are a good representation of the data.

Conclusions

So is there any meaning to Re_θ/M_e? When Re_θ/M_e is written out [Eq. (11)], it is seen that the U_e in the numerator and denominator cancel. Also, a_e and μ_e both depend only on external temperature.

$$Re_\theta/M_e = \rho_e U_e \theta/\mu_e(U_e/a_e) = \rho_e a_e \theta/\mu_e \tag{11}$$

For entry vehicles, these factors vary slowly with altitude. The density ρ_e, however, depends strongly on pressure and therefore varies very rapidly with altitude. Thus, Re_θ/M_e (or any Reynolds number criterion) might successfully correlate with the altitude at which transition occurs for a given entry vehicle.

The bottom line is that Re_θ/M_e does not represent any physical processes. It does not by itself give correct Mach number trends and it neglects pressure-gradient, surface-temperature, and roughness effects. Using Re_θ rather than Re_x tends to reduce data scatter, but the benefit is lost in applying Re_θ to get Re_x. For entry vehicles, Re_θ/M_e might work because density is the dominant factor in Re_θ/M_e and density is very sensitive to altitude. For cruise vehicles, that sensitivity is not relevant because one is interested in the x location of transition at constant altitude.

The proper analysis of transition behavior is by physics-based e^N or transient-growth methods. The use of correlations of questionable basis and reliability should be replaced by the physics-based methods.

Acknowledgments

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References

[1] Schmid, P. J., and Henningson, D. S., *Stability and Transition in Shear Flows*, Springer–Verlag, New York, 2001.
[2] Reshotko, E., "Transient Growth: A Factor in Bypass Transition," *Physics of Fluids*, Vol. 13, No. 5, 2001, pp. 1067–1075.
[3] Reshotko, E., and Tumin, A., "Role of Transient Growth in Roughness-Induced Transition," *AIAA Journal*, Vol. 42, No. 4, 2004, pp. 766–770.
[4] Andersson, P., Berggren, M., and Henningson, D., "Optimal Disturbances and Bypass Transition in Boundary Layers," *Physics of Fluids*, Vol. 11, No. 1, Jan. 1999, pp. 135–150.
[5] Schlichting, H., *Boundary Layer Theory*, 7th ed., McGraw–Hill, New York, 1979, pp. 540–542.
[6] Batt, R. G., and Legner, H. L., "A Review and Evaluation of Ground Test Data on Roughness Induced Nosetip Transition," TRW Defense and Space Systems Group, Rept. BMD-TR-81-58, Redondo Beach, CA, Oct. 1980.
[7] Batt, R. G., and Legner, H. L., "A Review of Roughness-Induced Nosetip Transition," *AIAA Journal*, Vol. 21, No. 1, 1983, pp. 7–22.
[8] Reda, D. C., "Correlation of Nosetip Boundary Layer Transition Data Measured in Ballistic Range Experiments," *AIAA Journal*, Vol. 19, No. 3, 1981, pp. 329–339.
[9] Reda, D. C., "Review and Synthesis of Roughness-Dominated Transition Correlations for Reentry Applications," *Journal of Spacecraft and Rockets*, Vol. 39, No. 2, 2002, pp. 161–167.
[10] Malik, M. R., "Prediction and Control of Transition in Supersonic and Hypersonic Boundary Layers," *AIAA Journal*, Vol. 27, No. 11, 1989, pp. 1487–1493.

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